Some useful operations and functions

Suppose you want to evaluate a function $f(t)$ at t-2. That you have an expression such as: $f(t) = 3 * t\text{^}5$ at t-2 and that you need, in your calculations, to replace $f(t)$ by $f(t-2) = 3*(t-2)\text{^}5$. You proceed as follows:

```
> h:=(3*u^5);
```

$$h := 3\,(t-2)^5$$

```
> u:=(t-2);
```

$$u := t-2$$

```
> h;
```

$$3\,(t-2)^5$$

(If necessary you can expand the above expression with the following command.)

```
> expand(%);
```

$$3\,t^5 - 30\,t^4 + 120\,t^3 - 240\,t^2 + 240\,t - 96$$

This procedure will save a lot of time in your calculations. The reason for bringing this on is that dealing with differential equations with discontinuous forcing functions the computations can become lengthy and tedious and in most of the practical problems, Machine use becomes inevitable. But then you should know how to use the machine to the fullest advantage.

Now that you have some idea of how to hand calculate Laplace transform of simple functions, it seems appropriate to introduce you to the commands that will make Maple give you the Laplace transform.

Suppose you want to find $L\,(t^3 + \sin(4t))$. You punch in the following command:

```
> laplace(t^3+sin(4*t)=y(t), t, s);
```

$$6\,\frac{1}{s^4} + \frac{4}{s^2+16} = \mathrm{laplace}(\mathrm{y}(t), t, s)$$

For other variations you can consult Maple help. There are enough examples there to allow me to just mention the topic. (You can search it from the glossary by clicking on “Mathematics” then (under Mathematics) on “Calculus” and then on integral transforms and finally under integral transforms you can find “laplace”. While I am at it, let me tell you that in the same column you can find “invlaplace” to get the inverse Laplace transform. Here is a simple command and for more you know where to look.

```
> invlaplace(6/s^4+4/(s^2+16),s,t);
```

$$t^3 + \sin(4\,t)$$

Indeed you can find the laplace transform of a differential equation such as:
$y'' - 2y' - 2y = 0, \quad y(0) = 2, \; y'(0) = 0$

Here is how:

```
> dsolve({diff(y(t),t$2)-2*diff(y(t),t)-
2*y(t)=0,y(0)=2,D(y)(0)=0},y(t),method=laplace);
```

$$\mathrm{y}(t) = \left(1 + \frac{1}{3}\sqrt{3}\right)\mathbf{e}^{(-(\sqrt{3}-1)\,t)} + \left(1 - \frac{1}{3}\sqrt{3}\right)\mathbf{e}^{((\sqrt{3}+1)\,t)}$$

Some times it may be hard to reconcile your Maple answers with those obtained by standard hand calculations, but I trust you would be able to resolve those issues. For instance the standard answer would be: $y = 2e^t \cosh(\sqrt{3}t) - (2/\sqrt{3})e^t \sinh(\sqrt{3}t)$.

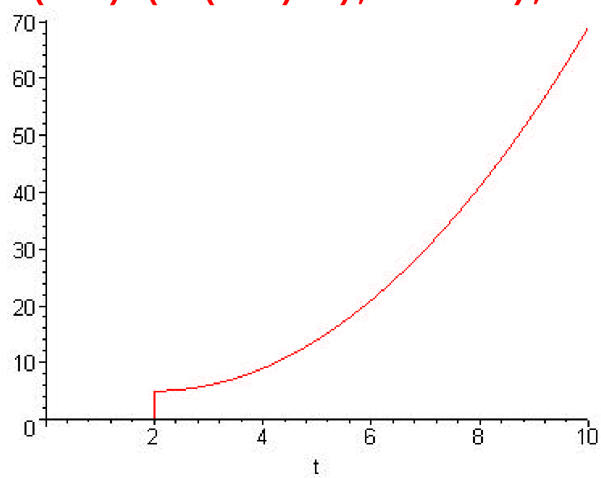
In case the forcing function is discontinuous usually the step -function called the "Heaviside function" is involved. You know that the Heaviside step-function is defined as $u_c(t) = \begin{cases} 0 & 0 \le t < c \\ 1 & t \ge c \end{cases}$ and that $u_c(t)\,f(t) = \begin{cases} 0 & 0 \le t < c \\ f(t) & t \ge c \end{cases}$ …. (1) and further that if you want to shift the graph of $y = f(t)$ to the right by $c>0$ you need to write $u_c(t)\,f(t-c) = \begin{cases} 0 & 0 \le t < c \\ f(t-c) & t \ge c \end{cases}$ ……(2). Notice that (1) and (2) have different graphs.

But before that you need to note that Maple does not recognize $u_c(t)$. Instead it requires you to input "Heaviside(t-c)" for $u_c(t)$. Below I give examples that would illustrate the use of Heviside function.

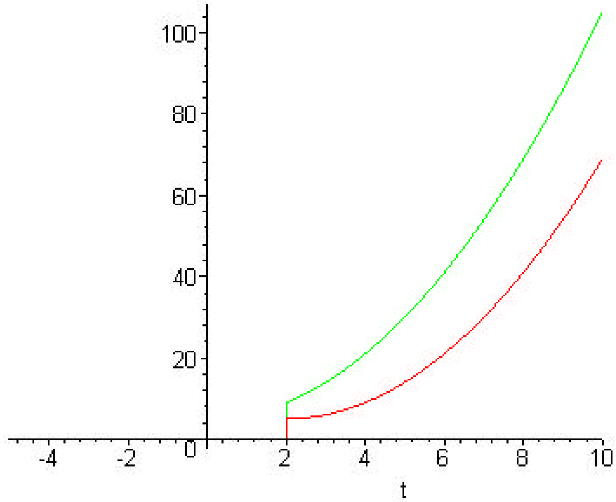
```
> laplace(Heaviside(t-2)*t,t,s);
```

$$2\,\frac{\mathbf{e}^{(-2\,s)}}{s} + \frac{\mathbf{e}^{(-2\,s)}}{s^2}$$

```
> plot(Heaviside(t-2)*(5+(t-2)^2),t=0..10);
```



```
> plot({Heaviside(t-2)*(5+t^2),Heaviside(t-2)*(5+(t-
2)^2)},t=-5..10);
```



>

Sometimes the differential equation is too long. In that case we use the following device: I have taken Example 2 from page 320 of your book.

The differential equation is: $y^{//}+4y=g(t)$, $y(0)=0$, $y^{(0)}=0$ where $g(t)$ is given by $g(t)=[u_5(t)(t-5)-u_{10}(t)(t-10)]/5$.

```
> de1:=diff(y(t),t$2)+4*y(t)=(Heaviside(t-5)*(t-5)-
Heaviside(t-10)*(t-10))/5;
```

$$de1 := \left(\frac{\partial^2}{\partial t^2}\mathrm{y}(t)\right)+4\,\mathrm{y}(t)=\frac{1}{5}\,\mathrm{Heaviside}\,(t-5)\,(t-5)-\frac{1}{5}\,\mathrm{Heaviside}\,(t-10)\,(t-10)$$

```
> dsolve({de1,y(0)=0,D(y)(0)=0},y(t),method=laplace);
```

$$\mathrm{y}(t)=\left(-\frac{1}{40}\sin(2\,t-10)+\frac{1}{20}\,t-\frac{1}{4}\right)\mathrm{Heaviside}\,(t-5)+\left(\frac{1}{2}+\frac{1}{40}\sin(2\,t-20)-\frac{1}{20}\,t\right)\mathrm{Heaviside}\,(t-10)$$

```
> plot(((-1/40)*sin(2*t-10)+(1/20)*t-1/4)*Heaviside(t-
5)+((1/2)+(1/40)*sin(2*t-20)-(1/20)*t)*Heaviside(t-10),
t=0..20);
```