MATH 360 Test 02, Spring 2002

Name:
Show all work and attach necessary printouts for computer-work.

1. Find (without using a computer algebra system or a calculator) the general solutions of each of the following differential equations.

(i). $y'' - 2y' - 3y = -3te^{-t} + 7t$

(ii). $y'' - 6y' + 9y = t^2 e^{3t}$

(iii). $y^{iv} + 8y'' + 16y = 0$

(iv). $y''' - y'' + y' - y = 0$

2. Verify that $x,\ x^2,\ 1/x$ are solutions of $x^3 y''' + x^2 y'' - 2xy' + 2y = 0$. Evaluate $W(x,\ x^2,\ 1/x)$ and decide if these solutions are linearly independent or not.

3. Solve the initial value problem: $2y'' + 2y' + y = 2,\ \ u(0) = -1,\ u'(0) = 1$, without a computer algebra system or a calculator, and respond to the following questions/requests.

(i). Describe the long-term behavior of the solution giving reasons.

(ii). Use the plot command to draw the graph of the solution over [0,15] and over [0,50]

(iii). Use your calculator to estimate the (time) intervals over which the solution is less than 2.

4. A series circuit has a capacitor of 10^{-5} farad, a resistor of 3×10^2 ohm, and an inductor of .2 henry. The initial charge on the capacitor is 10^{-6} coulomb and there is no initial current. Find the charge on the capacitor at any time t.

5. Solve the initial value problem: $4y'' + y = 0,\ \ u(0) = -1,\ u'(0) = 1$. Write the solution in the form $u = R\cos(w_0 t - d)$. If this initial value problem were to model a vibrating system, indicate the amplitude, the natural frequency, the period of the vibration and the phase angle.

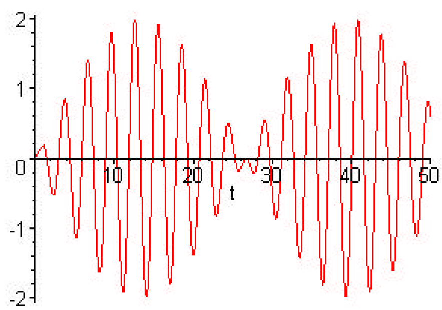
3. Match the initial value problems with the graphs of their solutions by either giving a good reason or actually producing the solutions and their graphs. Also, assuming that these initial value problems represent spring mass systems, label each as forced, unforced, damped, undamped etc.

(i). $u'' + 5u = \cos(2t), \quad u(0) = 0,\ u'(0) = 0$

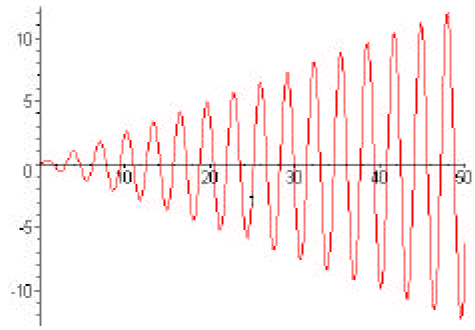
(ii). $u'' + 4u = \cos(2t), \quad u(0) = 0, u'(0) = 0$

(iii). $u'' + 4u' + 5u = 65\cos 2t,\ u(0) = 0, u'(0) = 1$

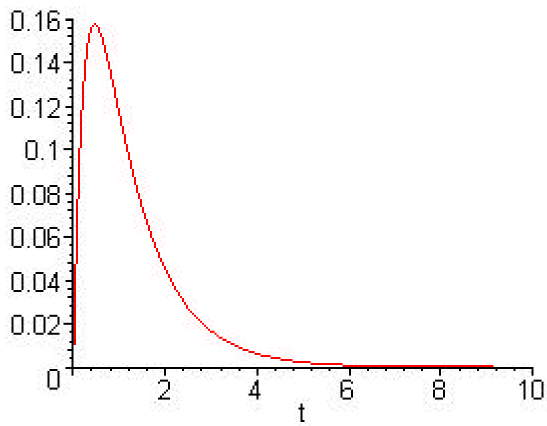
(iv). $u'' + 5u' + 4u = 0,\ u(0) = 0,\ u'(0) = 1$



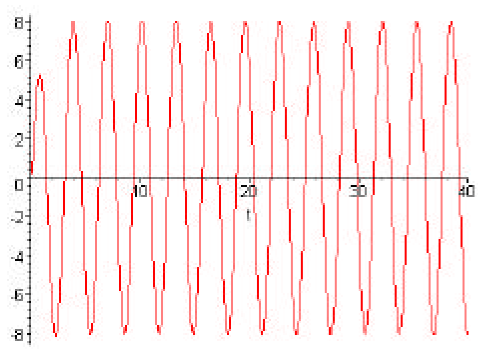
(I)



(II)



(III)



(IV)