MATH 360 Test 01(Pract.), Spring 2002

Name:

Support all answers with necessary, definitions, reasons and/or calculations.

1. A drug is absorbed by the body at a rate proportional to the amount *x(t)* present in the bloodstream. Assuming that any drug that enters the body intravenously is uniformly distributed in the whole body, set up a differential equation for the amount of the drug in the bloodstream of a patient if the drug is absorbed in at the rate of 0.4*x(t)* per hour and if the patient is simultaneously receiving the drug intravenously at the constant rate of 12 mg per hour.
2. Find the general solutions of the differential equations:

(i). $(1+t^2)y' + ty = (1+t^2)^{\frac{3}{2}}$

(ii) $y' = 1 - t + y^2 - ty^2$

3. Consider the initial value problem: $(4-t^2)y' + 2ty = 3t,\ y(0) = \frac{5}{2}$ and respond to the following.

(i). What is the longest interval over which this initial value problem is certain to have a solution?

(ii). Solve the initial value problem and sketch its graph over the interval you decided on in (i) above.

4. Find the general solution of the Bernoulli equation: $\frac{dx}{dt} + 12x = 24x^{\frac{3}{4}}$.
5. Of the following differential equations solve the one that is exact:

(i). $(2yx^2 + 2y)dx + (2xy^2 + 2x)dy = 0$

(ii). $((2x^2 + 2t + 1)dt + (4x^3 + 4tx)dx = 0$

6. Write down the general solutions of each of the following differential equations:

(i). $y'' - 2y' + y = 0$.

(ii). $2y'' - 2y' + y = 0$.

7. Find the Wronskian of the fundamental solutions y_1, y_2 of $ty'' + 2y' = 0,\ t > 0$ using Abel's Theorem. Obviously one of the solutions is a constant say d, what is the other?
8. Solve Problem 7 on page 58 of your book.
9. Solve the initial value problem: $15u'' - 2u' - u = 0,\ u(0) = u'(0) = 1$.