

MUHAMMAD ZAFRULLAH

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EDUCATION

- 1974 Ph. D. (Mathematics) University of London (Thesis Advisor: P. M. Cohn)
- 1969 M. Sc. (Mathematics) University of the Punjab Lahore, Pakistan
- 1967 B. Sc. (Mathematics and Physics) University of the Punjab, Lahore, Pakistan

PROFESSIONAL EXPERIENCE

- Fall 2000- Spring 2004 Visiting Assistant Professor, Idaho State University, Pocatello, ID
- 1999-Spr 2000 Lecturer, The University of Arkansas, Fayetteville, AR 72701
- 1998 Fall Visiting Associate Professor, University of Iowa, Iowa City , IA
- 1998 Spring Part-time instructor , Mount Mercy College, Cedar Rapids
- 1997 Fall Visiting Associate Professor, The University of Iowa, Iowa City, IA 52242.
- 1997 Spring Part-time Instructor, Bowie State University, Bowie, MD 20715.
- 1996 Fall Part-time Instructor, Bowie State University, Bowie, MD 20715.
- 1989-92 Associate Professor, Winthrop University, Rock Hill, SC 29733.
- 1989 Spring Visiting Associate Professor, The University of Iowa, Iowa City, IA 52242.
- 1988 Fall Visitor, Florida State University, Tallahassee, Florida 32306.
- 1987-88 Visitor, University of North Carolina at Charlotte, Charlotte, NC 28223..
- 1985-87 Honorary Research Fellow, University College London, London, UK.
- 1984 85 Associate Professor, Al- Faateh University, Tripoli, Libya.
- 1982- 84 Associate Professor, University of Sebha, Sebha, Libya
- 1977- 82 Assistant Professor, University of Sebha, Sebha, Libya
- 1975- 77 Demonstrator, UMIST, Manchester, UK.

PROFESSIONAL SERVICES

Reviewer, Mathematical Reviews, Zentralblatt MATH

Referee for Mathematical Journals

Outside reviewer of Ph.D. theses

Associate Editor, International Journal of Commutative Ring Theory

ADMINISTRATIVE EXPERIENCE

Chairman of the Department of Mathematics, University of Sebha, Sebha, Libya : from 1978 to 1983.

At Winthrop University I served on the Seminar Committee and the Winthrop-Wylie Mathematics Tournament Committee.

At the University of Arkansas, I was faculty adviser to a student organization called Pakistan Cultural Club.

PUBLIC SERVICE: I have started a web page to serve the following purposes: (a) to draw attention to the ever-increasing number of typographic and technical errors in undergraduate text-books, (b) to serve as a meeting place for Mathematicians working in Commutative Algebra. I have also started a helpdesk for multiplicative ideal theory. The web page has not attracted much attention yet, but I hope that it will flourish when there is a tendency of listening to what is being said regardless of who is saying it. I plan to use my web page to raise my voice for those who do not have a voice.

ABOUT THE USE OF TECHNOLOGY

I am familiar with, and sympathetic with, the use of technology in the classroom. In fact I have taught courses in computer labs, using Maple, Minitab and course/book specific software. My transition from the classical approach to the modern technological approach was not at all easy. I had to do a lot of hard thinking and arguing. Then in the end what converted me was the realization that the content of useful knowledge is increasing at a tremendous rate and to give our students at least a fighting chance of learning useful mathematics we must find some short cuts and use of technology is one of them. On my web page under ‘**Students**’ you can find notes on how to use TI-83 to do Statistics in the notes section of Math 253. These notes include pictures that show what the screen would look like when you press a certain button. Not only my students liked the notes, students from other schools have asked my (totally unnecessary) permission to use the notes.

SELECTED CONFERENCES AND VISITS

Conferences:

1. **Second Piedmont Mathematics Conference**, held at the University of North Carolina at Charlotte. (Duration: 3- 13- 87 to 3- 14- 87.) I gave a talk on the work that later developed into [24]. (A number in brackets such as this refers to the ordinal position of the paper in the list of my publications in My Research..)
2. **841st AMS meeting** at the University of Tennessee at Knoxville, TN. (Duration: 3- 24- 88 to 3-27- 88). Evan Houston gave a talk on [21].
3. **861st AMS meeting** held at Denton Texas in November 1990. Presented a paper on t-class groups and Nagata’s class group Theorem. The work later appeared as [47].

4. **Colloque International d'Algebre Commutative**, held at the University of Fez Morocco. (Duration: 4- 20- 92 to 4- 24- 92.) Gave a talk on t -invertibility especially highlighting the elements whose presence in an ideal ensures its invertibility (or t -invertibility). This work later appeared as [48].
5. **915th AMS Meeting** held at UT Chattanooga TN, in October 1997. -Gave a talk on unique factorization in non atomic rings
6. **936th AMS Meeting** held at Wake Forest University, Winston-Salem, NC in October, 1998.
7. **962nd AMS Meeting**, New Orleans, Louisiana, January 2001. Gave a talk on LCM-Splitting Sets in Some Ring Extensions, [68]
8. **International Conference on Commutative Ring Theory**, held at the Inha University in Incheon, Korea on May 18-19, 2001. Gave a talk on factorization and splitting sets. The material presented in this talk later developed into [74].
9. **Fourth International Conference, Commutative Ring Theory and Applications**, June 7 - 12 , 2001, held at Fez, Morocco. Gave a plenary talk on Various facets of rings between $D[X]$ and $K[X]$. Wrote a survey article with the same title. The survey "half appeared" in the conference proceedings, it has appeared in full in Comm. Algebra (see [71]). While in Fez I also served on the jury of Abdeslam Mimouni's thesis. Mimouni is a student of S. Kabbaj, a well-known Moroccan Algebraist.
10. **969th AMS Meeting**, September 21-23, 2001, Columbus, Ohio. I gave a talk on Another look at Nagata Type Theorems. In it I talk on t -LCM Splitting sets and corresponding Nagata type theorems. Most of this material went into [74].
11. **984th AMS Meeting**, March 14-16, 2003, Baton Rouge, Louisiana, gave a talk on $A+XB[X]$ domains and the HFD property. This is joint work with Jim Coykendall and Tiberiu Dumitrescu, the paper is submitted.
12. **994th AMS meeting**, March 12-13, 2004, Tallahassee, Fla., gave a talk on t -splitting sets of ideals and the influence Gilmer and Mott had on this area of research.
13. **Workshop on commutative rings and their modules**, May 30-June 4, 2004, Cortona, Italy, gave a plenary talk on v -coprimality and its applications. The title of the talk was, "What v -coprimality can do for you". I have written a survey article [85] on this topic.

Visits:

1. Marco Fontana of Istituto Matematico Universita di Roma invited me, in 1985, to visit for a week. Gave a talk discussed some Mathematics.
2. Alain Bouvier of Universite Claude Bernard, Lyon, France invited me, in 1986, to visit for a week or ten days. Gave several talks, discussed with Bouvier's students their future work.
3. Joe Mott of Florida State University, Tallahassee Florida invited me to visit and give a talk. This visit turned into a tour of several universities including, the University of Iowa, Iowa City, IA, the University of Tennessee at Knoxville, TN and the University of Virginia at Charlottesville, VA. I gave talks at all these places over a period of some twenty days and wrote or planned a number of papers.
4. While in Morocco, in June 2001, I visited Universite Qadi Ayyad, Marrakech, to serve as a member of jury on Said el Baghdadi's thesis. The main topic of the thesis was t -class groups. (El-Baghdadi, also, is a student of S. Kabbaj.) During my

stay there I discussed Mathematics with several people gave my preprints off my laptop and suggested some problems to a number of people.

REFERENCES

Professor Daniel Anderson, Department of Mathematics, The University of Iowa, Iowa City, IA 52242, USA. Phone: 319-335-0773, e-mail: dan-anderson@uiowa.edu

Professor Evan Houston, Department of Mathematics, University of North Carolina at Charlotte, Charlotte, NC 28213, USA. Phone: 704-547-2648, e-mail: eghousto@email.uncc.edu

Professor David Anderson, Department of Mathematics, The University of Tennessee, Knoxville, TN 37996 USA. Phone 865-974-4298, e-mail: anderson@math.utk.edu

MY RESEARCH

I have published a number of papers in Mathematical Journals of repute and I continue to be actively involved in research on (1) *Commutative Ring Theory*, (2) *Partially Ordered Groups* and (3) *Elementary Number Theory*. Ring Theory being my main area of interest I briefly describe what kind of work I have been doing in it.

In Ring Theory, I usually work on the following topics.

1. Non-Unique Factorization: An integral domain in which every nonzero non-unit is expressible as a product of finitely many irreducible elements is called an atomic domain. In this area we are interested in finding examples of atomic domains that are not UFD's (Unique Factorization Domains). We also try to see how far these atomic domains are from being UFD's. Here is a list of my papers on this topic (the numbers in square brackets represent the serial numbers of papers in the list of my publication at the end): [37], [38], [39], [42], [45], [62], [76], [81]. Some Mathematicians regard [37], [38] and [42] as the forerunners of the current activity in the theory of Non-unique factorizations. (If you are interested you may look up Geroldinger and Halter-Koch's book: Non-unique factorizations: algebraic, combinatorial and analytic theory, Chapman and Hall, CRC 2006.) My interest in atomicity, however goes back to [9] where I noted that atomicity under certain conditions implies unique factorization. Some signs of my interest can also be traced to [19] and to a problem in Cohn's [Algebra volume 2 (2nd Edition), John Wiley and Sons 1989, page 353].
2. Generalizations of Unique Factorization: Integral domains whose non-zero elements have some form of unique factorization. List of some of my publications

on this topic: [3], [6], [7], [12], [33], [34], [45], [54], [58], [59]. (My doctoral dissertation was on this topic.) Recently I have published [89] in collaboration with Stefania Gabelli and El-Baghdadi. This paper is on unique factorization of ideals.

3. Generalizations of UFD's: Generalizations of UFD's as locally finite intersections of localizations at height one primes. Of these the most well-known are the "weakly Krull" and the "weakly factorial" domains. Here is a list of some of my papers on this topic: [5], [12], [17], [19], [21], [33], [39], [43], [45], [54], [61], [69], [72]. To see the impact of some of my work in this area you may want to look up, "Ideal Systems, an introduction to Multiplicative Ideal Theory", by Franz Halter-Koch ISBN: 0-8247-0186-0. Halter-Koch has, in the above book, introduced notions such as weakly Krull monoids, weakly factorial monoids etc.
4. Polynomial Ring Constructions: Sub-rings of polynomial rings over fields (rings of the form $A + XB[X]$ where $A \subseteq B$ are subrings of a field K , X an indeterminate over K) to serve as examples. These pullback constructions became popular after the appearance of [4], [18], [28] and [38] from my list of publications. Today considerable literature exists on these constructions, you may consult papers by Tom Lucas and by Evan Houston and Stefania Gabelli that appear in the same book that contains my paper [64] as a chapter. The extent of my involvement in this area can be gauged from [71]. David Anderson Fontana, Kabbaj, Mimouni and Izlgue have contributed to these constructions a great deal. Recent papers that utilize this construction are: [81], [89].
5. Class Groups: The notion of the divisor class group is restricted to domains that are completely integrally closed and it is mostly used in the context of Krull domains. I suggested the notion of a class group, called the t -class group or simply the class group, to Alain Bouvier and answered his questions that helped him write [[Le groupe des classes d'un anneau integre, in « 107^{eme} Congres des Societes Savantes, Brest, \(1982\) Vol. 4, pp. 85-92.](#)]. This class group is defined for a general integral domain and it reduces to the divisor class group for a Krull domain. This notion is fast becoming popular among multiplicative ideal theorists. A number of articles have appeared on the study of class groups of various types of Domains. Among the contributors to this area David Anderson's name stands out, he has written at least two survey articles and given a four lecture course on class groups at a conference. (If I have it my way he would be writing a third and possibly a fourth survey article on this topic.) Other significant contributors are Fontana, Gabelli, Kabbaj, Mimouni, M.H. Park and G.W. Chang, and some more whose names I cannot remember. Geroldinger and Halter-Koch in their book on non-Unique Factorization (mentioned above) have introduced the class group via divisor theories. Halter Koch, in his book on multiplicative ideal theory (mentioned above) introduced the t -class group for a monoid in translation style. There has been a general misunderstanding that the notion of a t -class group is a generalization of the notion of the divisor class group. In a recent paper (preprint) David Anderson, Marco Fontana and I have given examples of completely integrally closed domains for which the divisor

class group is nonzero while the t -class group is zero. In connection with t -class groups I have written the following: [27], [33], [35], [40] and [47].

6. Study of star operations: The t -operation is one of the star operations that I have studied in detail. My article, "Putting t -invertibility to use" [64] may shed some light on my involvement in this area, the notion of the t -class group mentioned above depends upon the notion of t -invertibility, for a recent survey of the t -class groups and their applications Chapter 2 of "Non Noetherian Commutative Ring Theory, edited by Scott Chapman and Sarah Glaz, Kluwer Academic Publishers, Dordrecht/Boston/London, 2000. This is the same book where [64] appeared and Chapter 2 is written by David Anderson. In [64] I also discuss integral domains D whose nonzero finitely generated ideals are t -invertible. These are called Prufer v -multiplication domains (PVMD's). Joe Mott and I wrote, earlier, a useful paper [9] on this topic. Some more papers of mine on PVMD's and t -operations are: [10], [12], [17], [21], [24], [25], [26], [29], [39], [47], [48], [57], [66] and [89]. Other highlights of my work on t -operations are: (a) demonstrating in [18] and in [29] that if P is a prime t -ideal then PD_P may not be a prime t -ideal of D_P . This led to the notions of well-behaved prime ideals and of well-behaved domains. (b) In [21] we introduced the integral domains D such that every Upper to zero is a maximal t -ideal (UMT-domains). It turned out that D is a PVMD if and only if D is a UMT domain. Here an upper to zero is a prime ideal P in $D[X]$ such that $P \cap D = (0)$.
7. Divisibility: An integral domain D is a GCD domain if every pair of nonzero elements x, y in D have a Greatest Common Divisor (GCD) denoted often by $\text{GCD}(x, y)$. Paul Cohn [Proc. Cambridge Philos. Soc. 64(1968), 251- 264] called an integrally closed integral domain D a Schreier ring if for all x, y, z in $D \setminus \{0\}$ $x | yz$ in D implies $x = rs$ where $r | y$ and $s | z$ and showed that a GCD domain is a Schreier domain. In [4] we (me and my co-authors) showed that if D is a GCD domains and S is a multiplicative set in D then the construction $D + XD[1/S] [X]$ is a Schreier domain. I subsequently studied pre-Schreier domains in [16] and have written the following papers on (pre-) Schreier domains and related topics: [18], [29], [50], [53], [63], [83], [84].
8. Almost GCD domains: Almost GCD (AGCD) domains are integral domains D with the property that for each pair of nonzero elements x, y in D there is a natural number $n(x,y)$ such that the intersection of the ideals (x^n) and (y^n) is principal. I introduced this notion in [12]. (Special cases are Dedekind domains with torsion class groups and Krull domains with torsion divisor class groups.) I also showed that an integrally closed AGCD domain is a PVMD with torsion class group. Note that if for each pair x, y as above $n(x, y) = 1$ the resulting AGCD domain is just the GCD domain. From AGCD domains Dan Anderson and I went to Almost Bezout domains in [35] (and much more) where we also showed that the t -class group of an AGCD domain is torsion. Dan Anderson and some of his students produced some research right after [35]. Recently David Anderson and G.W. Chang, have produced some research on this topic. Mimouni and Badawi have also done some interesting work. My recent papers in this connection are: [68], [69], [73] and [78].

RECENT AND CURRENT WORK:

In my study of a special case of rings of the form $A+XB[X]$ where B is a quotient ring of A , I have recently shown that if A is a Prufer v -multiplication domain $A+XA[I/S][X]$ is a Prufer v -multiplication domain (PVMD) if and only if for every nonzero d in A , the ideal (d, X) is t -invertible. Here, A is a PVMD if every finitely generated nonzero ideal I of A is t -invertible, i.e. there is a finitely generated ideal J such that $A: (IJ)=A$. The fall out from this study is an interesting set of results. I have written two papers on this with Dan Anderson. The titles of these two papers are: (a) **Splitting multiplicative sets** (This paper has appeared ([63]) in Proc. Amer. Math. Soc. (b) **The ring $R+XR[I/S][X]$ and t -splitting sets.** (This paper has appeared ([66]) in Arabian Journal of Science and Engineering.) The t -splitting sets and their variations have in fact been quite useful an interested reader may look up [80] and [81].

Let me recall that two elements x, y of an integral domain D are v -coprime (or LCM coprime) if $xD \cap yD = xyD$. Recall also that a saturated multiplicative set S of an integral domain D is a splitting set if every nonzero non-unit x of D can be written as $x = sd$ where $s \in \hat{S}$ and d is v -coprime to every element of S . Moreover a splitting set is LCM-splitting if in addition for each $s \in \hat{S}$ we have $sD \cap xD$ is principal for each x in D . The splitting multiplicative sets have been quite useful in studying the multiplicative properties of integral domains. For instance, in [18] I showed that if D is a GCD domain and S is a multiplicative set in D , then $D+XD[I/S][X]$ is a GCD domain if and only if S is a splitting multiplicative set of D . In [63] some new characterizations were added and in [66] a new notion of splitting sets was introduced.

One of my continuing interests is Nagata type Theorems (If S is an LCM splitting set and the ring of fractions $D[I/S]$ is a PVMD (GCD domain) then so is D .) Recently, in a paper with Tiberiu Dumitrescu, I studied the question of how splitting sets and LCM splitting sets behave in some ring extensions. An interesting consequence of the study was that if D is a Noetherian domain and S is a multiplicative set of D generated by principal primes, then the integral closure of D is a UFD if and only if the integral closure of $D[I/S]$ is a UFD. (This is a new addition to the list of Nagata type theorems.). This paper ([68]) has electronically appeared in Proc. Amer. Math. Soc. in November 2001.

I have recently started writing a survey of Nagata type Theorems with D.D. Anderson. My interest in the study of almost GCD domains also continues. (Almost GCD domains are integral domains D with the property that for each pair of elements x, y in D there is a natural number $n(x, y)$ such that the intersection of the ideals (x^n) and (y^n) is principal.) Recently I have completed a study of almost GCD domains of finite character in collaboration with T. Dumitrescu, Y. Lequain and J. Mott. The paper ([69]) has appeared in J. Algebra. My interest in almost GCD domains continues and at present I am working with other mathematicians on the condition under which polynomial extensions of almost GCD domains are again almost GCD domains. Also of interest to me is the study of what I like to call almost splitting sets: These are saturated multiplicative sets S of a domain D

such that for each nonzero nonunit x of D there exists $n=n(x)$ such that x^n is expressible as a product $x^n=sd$ where $s \in S$ and d is v -coprime to every member of S .

Another ambition of mine is to produce Nagata type theorems that do not have anything to do with GCD or pre-Schreier property. I hinted at the possibilities in my talks at Inha University and at the Columbus Ohio meeting. ([74] and a paper under preparation resulted from these talks.)

Some time ago I studied an arithmetical function F , that has a multiplicative companion f such that for coprime natural numbers m,n $F(mn)=(F(m))^f(n)(F(n))^f(m)$. I called F a generalized multiplicative function. I am now working on this function to explore its utility. This function being somewhat accessible to undergraduates, I am trying to attract some undergraduates to work on it.

In [69] we floated the idea of an almost lattice ordered group. I am considering plunging into partially ordered groups to get some non-abelian examples of almost lattice ordered groups. I have recently written a paper on factoriality in Riesz groups, with Joe Mott and Muneer Rashid [88] (A Riesz group, incidentally, is a directed partially ordered group that satisfies Riesz interpolation property.)

An integrally closed integral domain D is called a v -domain if every finitely generated nonzero ideal of D is v -invertible, i.e. for every finitely generated ideal A we have $(AA^{-1})_v = D$. In a recent paper, with Anderson, Anderson and Fontana, we study integral domains D in which every finitely generated nonzero ideal is $*$ -invertible and call these domains $*$ -Prüfer.

MY GENERAL ATTITUDE TOWARDS RESEARCH

My general attitude towards research is best described by saying that I chase patterns. When I see a good result I want to know what is happening behind the scenes, that causes this result and once I find the facts that caused the beautiful result I indicate how else those facts can be used. Most of my papers are good examples of that but my work on Nagata like theorems and on Kaplansky like theorems stands out. Here Nagata like Theorems are statements that generalize Nagata's theorem on UFD's: Let R be an integral domain that satisfies ACC on principal ideals, and let S be a multiplicative set generated by primes of R . If R_S is a UFD then so is R . Next, the theorem of Kaplansky that I targeted is: An integral domain R is a UFD if and only if, every nonzero prime ideal of R contains a nonzero principal prime. I have singled out these two theorems because I was so impressed by them that I actually went chasing their patterns into partially ordered groups (not necessarily abelian) and came up with some interesting results.

RESEARCH PUBLICATIONS

- [1]. A note on two generated finite groups with two defining relations, Punjab Univ. J. Math. (Lahore) 4(1971), 67-68.
- [2]. On the evaluation of a certain arithmetical function, J. Natur. Sci. and Math. 12(1972), 363-365 (with S.M. Kerawala).
- [3]. Semirigid GCD-domains, Manuscripta Math. 17(1975), 55-66.
- [4]. The construction $D + XD_S[X]$, J. Algebra 53(1978), 423-439 (with D.L. Costa and J.L. Mott).
- [5]. On a result of Gilmer, J. London Math. Soc. 16 (1977), 19-20.
- [6]. Rigid elements in GCD domains, J. Natur. Sci. and Math. 17(1977), 7-14.
- [7]. On unique representation domains, J. Natur. Sci. and Math. 18(1978), 19-29.
- [8]. On finite conductor domains, Manuscripta Math. 24(1978), 191-204.
- [9]. On Prüfer v -multiplication domains, Manuscripta Math. 35(1981), 1-26 (with J.L. Mott).
- [10]. Some polynomial characterizations of Prüfer v -multiplication domains, J. Pure Appl. Algebra 32 (1984), 231-237.
- [11]. The v -operation and intersections of quotient rings of integral domains, Comm. Algebra 13(1985) 1699-1712.
- [12]. A general theory of almost factoriality, Manuscripta Math. 51(1985), 29-62.
- [13]. Overrings and dimensions of general $D + M$ constructions, J. Natur. Sci. and Math. 26(2) (1986), 7-14 (with D.L. Costa and J.L. Mott).
- [14]. The GCD property and irreducible quadratic polynomials, International J. Math. 9(1986), 749-752 (with S.B. Malik and J.L. Mott).
- [15]. On generalized Dedekind domains, Mathematika 33(1986), 285-296.
- [16]. On a property of pre-Schreier domains, Comm. Algebra 15(1987), 1895-1920.
- [17]. On t -invertibility, Comm. Algebra 16(1988), 149-170 (with S.B. Malik and J.L. Mott).
- [18]. The $D + XD_S[X]$ construction from GCD-domains, J. Pure Appl. Algebra 50(1988), 93-107.
- [19]. Two characterizations of Mori domains, Math. Japonica 33(1988), 645-652.
- [20]. On generalized multiplicative functions I, J. Natur. Sci. and Math. 28(1988), 257-268.
- [21]. Integral domains in which each t -ideal is divisorial, Michigan Math. J. 35(1988), 291-300 (with E. Houston).
- [22]. Ascending chain conditions and star operations, Comm. Algebra 17(6) (1989), 1523-1533.
- [23]. Some characterizations of v -domains and related questions, Colloq. Math. Vol. LVIII (1989), 1-9 (with D.D. Anderson, D.F. Anderson, D. Costa, D. Dobbs and J.L. Mott).
- [24]. Some quotient based characterizations of domains of multiplicative ideal theory, Bull. Math. Ital. (7) 3-B (1989), 455-476 (with D.D. Anderson and J.L. Mott).
- [25]. On t -invertibility II, Comm. Algebra 17(8) (1989), 1955-1969 (with E. Houston).
- [26]. t -linked overrings and Prüfer v -multiplication domains, Comm. Algebra 17(11)(1989), 2635-2852 (with D. Dobbs, E. Houston and T. Lucas).
- [27]. On some class groups of an integral domain, Bull. Soc. Math. Grece. 29(1988), 45-59 (with A. Bouvier).
- [28]. Unruly Hilbert domains, Canad. Bull. Math. 33(1) (1990), 106-109 (with J.L. Mott).

- [29]. Well behaved prime t -ideals, *J. Pure Appl. Algebra* 65(1990), 199-207.
- [30]. Contents of polynomials and invertibility, *Comm. Algebra* 18(5) (1990), 1569-1583 (with J.L. Mott and B. Nashier).
- [31]. Flatness and invertibility of ideals, *Comm. Algebra* 18(7)(1990), 2151-2158.
- [32]. t -linked overrings as intersections of localizations, *Proc. Amer. Math. Soc.* 109(3)(1990), 637-646 (with D. Dobbs, E. Houston and T. Lucas).
- [33]. Weakly factorial domains and groups of divisibility, *Proc. Amer. Math. Soc.* 109(4)(1990), 907-913. (with D.D. Anderson).
- [34]. Factoriality in partially ordered groups, *Comm. Algebra* 18(5)(1990), 1307-1322.
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- [37]. Factorization in integral domains, *J. Pure Appl. Algebra* 69(1990), 1-19 (with D.D. Anderson and D. F. Anderson).
- [38]. Rings between $D[X]$ and $K[X]$, *Houston J. Math.* 17(1)(1991), 109-129 (with D.D. Anderson and D.F. Anderson).
- [39]. On Krull domains, *Archiv der Math.* 56(1991), 559-568 (with J.L. Mott).
- [40]. Splitting the t -class group, *J. Pure Appl. Algebra* 74(1991), 17-37 (with D.D. Anderson and D.F. Anderson).
- [41]. t -linked overrings of Noetherian weakly factorial domains, *Proc. Amer. Math. Soc.* 115(3)(1992), 601-604 (with M. Martin).
- [42]. Factorization in integral domains II, *J. Algebra* 152(1992), 78-93 (with D.D. Anderson and D.F. Anderson).
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- [48]. On t -invertibility and comparability, *Commutative Ring Theory* (eds. P.-J. Cahen, D. Costa, M. Fontana and S.-E. Kabbaj), Marcel Dekker, New York, 1994, 141-150 (with R. Gilmer and J. Mott).
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- [51]. A note on triangular numbers, *Punjab. Univ. J. Math.* 26(1993), 75-83 (with H. Lee).
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- [53]. P.M. Cohn's completely primal elements, *Zero-Dimensional Commutative Rings* (eds. D.F. Anderson and D. Dobbs) Marcel Dekker, New York, 1995, 115-123 (with D.D. Anderson).

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- [87] Monoid domain constructions of antimatter domains (with Dan Anderson, Linda Hill and Jim Coykendall) (to appear in *Comm. Algebra*).
- [88] Factoriality in Riesz groups (with Joe Mott and Muneer Rashid) to appear in *J. Group Theory*.
- [89] Unique representation domains II (with Said El-Baghdadi and S. Gabelli) to appear in *J. Pure Appl. Algebra*.

WORK IN PROGRESS.

- [1] Nagata Type Theorems a survey (with Dan Anderson) (Dormant)
- [2] Generalized multiplicative functions. (Dormant).
- [3] Unrestricted UFD's (with J. Coykendall) (in preparation).
- [4] Some remarks on PVMD's and class groups (with D.F. Anderson and M. Fontana) (submitted)
- [5] A simple application of Zorn's Lemma (with David Dobbs)
- [6] Almost Bezout domains, III (with Dan Anderson)

- [7] On \ast -Completely integrally closed domains and \ast -Prüfer domains (with DD and DF Anderson and M. Fontana) in preparation.
- [8] t -Schreier domains (with Tiberiu Dumitrescu) in preparation.