QUESTION: (HD0703) Is it correct to define a Prüfer domain as a domain whose finitely generated ideals are invertible?

ANSWER: Not entirely. The proper definition is: An integral domain D is a Prüfer domain if and only if every finitely generated nonzero ideal of D is invertible. The reason is that the zero ideal is not invertible. However the the definition you have quoted often gets used, in research articles, for any of the following reasons.

1. A ring whose finitely generated ideals are projective is called a semihereditary ring and a semihereditary integral domain is a Prüfer domain.

2. A ring whose finitely generated, and hence all, ideals are flat is a domain with weak global dimension at most one and a domain with weak global dimension at most one is a Prüfer domain.

Now the trouble is that in the above two cases the zero ideal is free and hence projective (and flat) and those who usually work with homological concepts are prone to translate directly and say that in a Prüfer domain every finitely ideal is invertible.

3. An integral domain D is a Bezout domain if every finitely generated ideal of D is principal (correct because the zero ideal is principal) and a Prüfer domain is a generalization of a Bezout domain. So some people unthinkingly genralize the definition of Bezout domains by changing "principal" to "invertible".

4. Some folks may call a Prüfer domain as a domain whose finitely generated ideals are invertible assuming that finitely generated means nonzero finitely generated.

Whatever the reasons it does not hurt to be exact. Let me end my answer by quoting the definition from page 38 of Kaplansky's book "Commutative Rings" Allyn and Bacon, 1970. This indicates that Kaplansky too was concerned about the "translating" bit that I mentioned.

"Definition. A Prüfer domain is an integral domain in which every non-zero finitely generated ideal is invertible. (Recall that a Bézout domain is slightly more special in that every finitely generated ideal is required to be principal.)"